Mat 2377

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Solution 1 on (32)

1. 12 ($2points$)

a) $M \cup N = \{x|0 < x < 9\}$

b) $M \cap N = \{x|1 < x < 5\}$

c) $M' \cap N' = \{x|9 < x < 12\}$

1.24 ($2points$)The multiplication principle yields the answer $2{\times}2 \times ... \times 2 = 2^9 = 512$

1.30 ($6points$)

(a) Any of the 6 nonzero digits can be chosen for the hundreds position, and of the remaining 6 digits for the tens position, leaving 5 digits for the units position. So, there are $(6)(6)(5) = 180$ three digit numbers.

(b) The units position can be filled using any of the 3 odd digits. Any of the remaining 5 nonzero digits can be chosen for the hundreds position, leaving a choice of 5 digits for the tens position. By Theorem 2.2, there are $(3)(5)(5) = 75$ three digit odd numbers.

(c) If a 4, 5, or 6 is used in the hundreds position there remain 6 and 5 choices, respectively, for the tens and units positions. This gives $(3)(6)(5) = 90$ three digit numbers beginning with a 4, 5, or 6. If a 3 is used in the hundreds position, then a 4, 5, or 6 must be used in the tens position leaving 5 choices for the units position. In this case, there are $(1)(3)(5) = 15$ three digit number begin with a 3. So, the total number of three digit numbers that are greater than 330 is $90 + 15 = 105$.

1.40 ($4points$)

(a) Let A = Defect in brake system; B = Defect in fuel system;

$\mathrm{P}(\mathrm{A} \bigcup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A} \bigcap \mathrm{B}) = 0.25 + 0.17 - 0.15 = 0.27.$

(b) $\mathrm{P}(\text{No defect}) = 1 - \mathrm{P}(\mathrm{A} \bigcup \mathrm{B}) = 1 - 0.27 = 0.73.$

1.48 ($2points$) a) $1 - 0.42 = 0.58$ b) $1 - 0.04 = 0.96$

1.50 $(4points)$ a) $0.02+0.30=0.32$
b) $0.32+0.25+0.30=0.87$
c) $0.05+0.06+0.02=0.13$
d) $1-0.05-0.32=0.63$
1.60 $(2points)$a) $0.90(0.08)=0.072$
b) $0.90(0.92)(0.12)=0.099$
1.72 $(2points)$ Let S_i be the event that a person speeds as he passes location L_i and let R be the event that the radar traps is operating resulting in a ticket. Then the probability that he receives a speeding ticket is

$$\begin{aligned} P(R) &= \sum P(R|S_i)P(S_i) \\ &= 0.4(0.2)+0.3(0.1)+0.2(0.5)+0.3(0.2)=0.27 \end{aligned}$$

1.92 $(6points)$Let A be the event that two non defective components are selected and let N be the event that a lot does not contain defective components. Then

$$P(N)=0.6, P(A|N)=1$$

Let O be the event that a lot containes one defective component and let T be the event that a lot contains two defective components. Then

$$P(O)=0.3, P(A|O)=\frac{\binom{19}{2}}{\binom{20}{2}}=\frac{9}{10}$$

and

$$P(T)=0.1, P(A|T)=\frac{\binom{18}{2}}{\binom{20}{2}}=\frac{153}{190}$$

a) $P(N|A)=\frac{P(A|N)P(N)}{P(A|N)P(N)+P(A|O)P(O)+P(A|T)P(T)}=\frac{0.6}{0.9505}=0.6312$
b) $P(O|A)=\frac{(9/10)0.3}{0.9505}=0.2841$
c) $P(T|A)=1-0.6312-0.2841=0.0847$
1.94 $(2points)$ let D be the event that an item is defective.
a) $P(D_1\cap D_2\cap D_3)=P(D_1)P(D_2)P(D_3)=0.2^3=0.008$
b) $P(\text{three out of four are defectives})=\binom{4}{3}(0.2)^3(1-0.2)=0.0256$